第27卷 第2期 工 程 数 学 学 报 Vol. 27 No. 2
2010年04月 CHINESE JOURNAL OF ENGINEERING MATHEMATICS Apr. 2010

文章编号:1005-3085(2010)02-0283-12

时滞模糊奇异摄动系统 H_∞ 滤波*

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摘 要: 针对一类变时滞非线性奇异摄动系统，本文提出了基于T-S模糊模型的建模方法，并设计了模糊 H_∞ 滤波器，通过求解一组与摄动参数 ε 无关的线性矩阵不等式，获得其增益，避免了由 ε 引起数值求解的病态问题，所获得滤波器使闭环系统渐进稳定且达到给定的 H_∞ 性能指标。该方法适用于标准和非标准非线性奇异摄动系统，仿真实例说明该方法的有效性。

关键词: 奇异摄动；时滞；H_∞ 滤波；线性矩阵不等式 (LMIs)；T-S 模型

分类号: **AMS**(2000) 93C40 **中图分类号**: TP13 **文献标识码**: A

1 引言

多时标现象广泛存在于复杂电子电路、柔性机器人、通信网络等各种物理、化学及人工系统中。作为处理由多时标现象引发病态问题的理想工具，奇异摄动技术得到了深入研究[1-5]。但目前传统方法研究非线性奇异摄动系统[6-9]，对系统结构的假设还较多。近来，模糊控制理论与非线性奇异摄动系统相结合的研究，较好地利用线性奇异摄动系统相关理论，解决了非时滞模糊奇异摄动系统分析与控制问题[10-15]，但时滞模糊奇异摄动系统的研究尚不多见[16]。文献 [16] 利用LMI方法研究了时滞模糊奇异摄动系统的稳定性条件和 H_2 状态反馈控制器问题。本文建立连续时滞模糊奇异摄动系统模型，并采用LMI方法研究了一类时滞模糊奇异摄动系统不依赖于摄动参数的 H_∞ 滤波器设计问题。该方法无需快慢分解且适用于标准和非标准非线性奇异摄动系统，为研究非线性时滞奇异摄动系统的 H_∞ 控制提供新思路。

2 时滞模糊奇异摄动系统模型

在本节中，我们参考文献 [16]，建立时滞模糊奇异摄动系统模型，并研究时滞非线性奇异摄动系统的 H_∞ 滤波问题。

时滞模糊奇异摄动系统第 $i\,(1\leqslant i\leqslant r)$ 条规则为如下形式。

Plant Rule i: If $\xi_1(t)$ is F_{i1}, and $\cdots$ and $\xi_g(t)$ is F_g, then

$$\begin{aligned}
E(\varepsilon)\dot{x}(t) &= A_i x(t) + A_{di}x(t-\tau(t)) + B_{1i}w(t),\\
z(t) &= C_{1i}x(t), \qquad\qquad (1)\\
y(t) &= C_{2i}x(t) + D_{21i}w(t),
\end{aligned}$$

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***基金项目**: 国家高新技术产业化专项项目 (2005-1).

其中

$$E(\varepsilon)=\begin{bmatrix} I_{n\times n} & 0 \\ 0 & \varepsilon I_{m\times m} \end{bmatrix},\quad x(t)=\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix},$$

且$x_1(t)\in\mathbf{R}^n$，$x_2(t)\in\mathbf{R}^m$为慢、快状态向量，$w(t)\in\mathbf{R}^p$为噪声信号(包括过程和测量噪声)，$y(t)\in\mathbf{R}^h$为测量输出，$z(t)\in\mathbf{R}^l$为待估计的信号向量，$F_{ij}\ (j=1,2,\cdots,g)$为模糊集合，r为规则数目，A_i, A_{di}, B_{1i}, C_{1i}, C_{2i}, D_{21i}为适当维数的矩阵，$\xi_1(t),\cdots,\xi_g(t)$为可测变量。$\tau(t)\le\tau_0$为有界时变时滞，且$\dot\tau(t)\le\beta<1$，其中τ_0和β均为已知常数。$0<\varepsilon\ll 1$为摄动参数。

采用标准的模糊推理方法一即单点模糊化，乘积推理，和加权平均清晰化，得全局模糊模型为

$$\begin{aligned} E(\varepsilon)\dot x(t)&=A(\mu)x(t)+A_d(\mu)x(t-\tau(t))+B_1(\mu)w(t),\quad x(0)=0,\\ z(t)&=C_1(\mu)x(t),\\ y(t)&=C_2(\mu)x(t)+D_{21}(\mu)w(t), \end{aligned}\tag{2}$$

其中

$$\mu_i=\mu_i(\xi(t)),\quad \mu_i(\xi(t))=\frac{w_i(\xi(t))}{\sum\limits_{i=1}^{r}w_i(\xi(t))},\quad w_i(\xi(t))=\prod_{j=1}^{g}F_{ij}(\xi_j(t)),$$

$$x(t)\in\mathbf{R}^{(n+m)},\quad x_d(t-\tau(t))\in\mathbf{R}^{(n+m)},\quad A(\mu)=\sum_{i=1}^{r}\mu_iA_i,$$

$$A_d(\mu)=\sum_{i=1}^{r}\mu_iA_{di},\quad B_1(\mu)=\sum_{i=1}^{r}\mu_iB_{1i},\quad C_1(\mu)=\sum_{i=1}^{r}\mu_iC_{1i},$$

$$C_2(\mu)=\sum_{i=1}^{r}\mu_iC_{2i},\quad D_{21}(\mu)=\sum_{i=1}^{r}\mu_iD_{21i},\quad E(\varepsilon)=\begin{bmatrix} I_{n\times n} & 0 \\ 0 & \varepsilon I_{m\times m} \end{bmatrix}.$$

对于系统(2)，如果应用常规模糊系统理论和Lyapunov方法，可得出一组依赖于摄动参数ε的稳定性条件。但由于$\varepsilon\ll 1$，系统矩阵会存在严重的病态特性。而目前通用的一些LMI求解工具，对矩阵的条件数均较为敏感[17]，故这类病态LMIs不适合用常规LMI工具求解。对稳定性分析问题如此，对综合问题也不例外。

下面提出时滞系统(2)不依赖于摄动参数ε的稳定性分析与H_∞滤波器设计方法。

3 H_∞滤波器设计

假设：时滞系统(2)是渐进稳定的。

对于时滞系统(2)，采用式(4)的控制规律，设计H_∞滤波器。

控制器规则i: If $\xi_1(t)$ is F_{i1}, and $\cdots$ and $\xi_g(t)$ is F_g, then

$$\begin{aligned} E(\varepsilon)\dot{\hat x}(t)&=A_{fi}\hat x(t)+B_{fi}y(t),\\ \hat z(t)&=C_{fi}\hat x(t). \end{aligned}\tag{3}$$

利用标准模糊推理方法，得整体控制器为

$$
\begin{aligned}
E(\varepsilon)\dot{\hat{x}}(t) &= A_f(\mu)\hat{x}(t) + B_f(\mu)y(t),\\
\hat{z}(t) &= C_f(\mu)\hat{x}(t),
\end{aligned}
\tag{4}
$$

其中

$$
A_f(\mu) = \sum_{i=1}^{r}\mu_i A_{fi},\quad B_f(\mu) = \sum_{i=1}^{r}\mu_i B_{fi},\quad C_f(\mu) = \sum_{i=1}^{r}\mu_i C_{fi},
$$

$$
\hat{x}(t) = \begin{bmatrix} \hat{x}_1(t) \\ \hat{x}_2(t) \end{bmatrix},
$$

$\hat{x}_1(t) \in \mathbf{R}^n$, $\hat{x}_2(t) \in \mathbf{R}^m$ 分别为滤波器的慢、快状态向量。

将控制器 (4) 应用到系统 (2)，得到闭环系统 (5)

$$
\begin{aligned}
E_{cl}(\varepsilon)\dot{\tilde{x}}(t) &= A_{cl}(\mu)\tilde{x}(t) + A_{dcl}(\mu)\tilde{x}(t-\tau(t)) + B_{cl}(\mu)w(t),\quad \tilde{x}(0)=0,\\
\tilde{z}(t) &= C_{cl}(\mu)\tilde{x}(t),
\end{aligned}
\tag{5}
$$

其中

$$
A_{cl}(\mu) = \begin{bmatrix} A(\mu) & 0 \\ B_f(\mu)C_2(\mu) & A_f(\mu) \end{bmatrix},\quad B_{cl}(\mu) = \begin{bmatrix} B_1(\mu) \\ B_f(\mu)D_{21}(\mu) \end{bmatrix},
$$

$$
\tilde{x}(t) = \begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix},\quad A_{dcl}(\mu) = \begin{bmatrix} A_d(\mu) & 0 \\ 0 & 0 \end{bmatrix},\quad E_{cl}(\varepsilon) = \begin{bmatrix} E(\varepsilon) & 0 \\ 0 & E(\varepsilon) \end{bmatrix},
$$

$$
\tilde{x}(t-\tau(t)) = \begin{bmatrix} x(t-\tau(t)) \\ 0 \end{bmatrix},\quad \tilde{z}(t) = z(t) - \hat{z}(t).
$$

定理 1　对于任意给定常数 $\gamma > 0$ 和 $\varepsilon \in (0, \varepsilon^*]$ $(0 < \varepsilon^* \ll 1)$，时滞模糊奇异摄动系统 (2) 存在一个 γ-次优 H_∞ 滤波器 (4) 当且仅当存在矩阵

$$
X = \begin{bmatrix} X_{11} & 0 \\ X_{21} & X_{22} \end{bmatrix},\quad Y = \begin{bmatrix} Y_{11} & 0 \\ Y_{21} & Y_{22} \end{bmatrix},
$$

$X_{11} \in \mathbf{R}^{n\times n}$, $X_{22} \in \mathbf{R}^{m\times m}$, $Y_{11} \in \mathbf{R}^{n\times n}$, $Y_{22} \in \mathbf{R}^{m\times m}$ 分别为对称正定矩阵，对称正定矩阵 $Q_{11} \in \mathbf{R}^{(n+m)\times(n+m)}$, $\hat{Q}_{22} \in \mathbf{R}^{(n+m)\times(n+m)}$ 和适当维数矩阵 M_i, N_i, J，使得矩阵不等式 (6)-(9) 成立。进而，如果矩阵不等式 (6)-(9) 有可行解，则可以按式 (10) 构造系统 (2) 的 γ-次优 H_∞ 滤波器。(下面式中*号表示对称项。)

$$
\Pi_{ii} < 0,\quad i = 1, 2, \cdots, r, \tag{6}
$$

$$
\Pi_{ij} + \Pi_{ji} < 0,\quad i < j < r,\quad i, j = 1, 2, \cdots, r, \tag{7}
$$

$$
X_{11} - Y_{11} > 0, \tag{8}
$$

$$
X_{22} - Y_{22} > 0, \tag{9}
$$

$$\Pi_{ij} = \begin{bmatrix} \Delta_{11} & * & * & * & * & * \\ \Delta_{21} & \Delta_{22} & * & * & * & * \\ B_{1i}^T Y & B_{1i}^T X + D_{21i}^T J_j^T & -I & * & * & * \\ C_{1i} - N_j & C_{1i} & 0 & -\gamma^2 I & * & * \\ A_{di}^T & A_{di}^T X & 0 & 0 & -(1-\beta)Q_{11} & * \\ 0 & 0 & 0 & 0 & 0 & -(1-\beta)\hat{Q}_{22} \end{bmatrix},$$

其中

$$\begin{aligned} &\Delta_{11} = A_i^T Y + Y^T A_i + Q_{11} + \hat{Q}_{22}, \\ &\Delta_{21} = A_i^T Y + X^T A_i + J_j C_{2i} + M_j + Q_{11}, \\ &\Delta_{22} = A_i^T X + X^T A_i + J_j C_{2i} + (J_j C_{2i})^T + Q_{11}, \\ &A_{fi} = (Y-X)^{-1} M_i, \quad B_{fi} = (Y-X)^{-1} J_i, \quad C_{fi} = N_i. \end{aligned} \tag{10}$$

证明 对于充分小的$\varepsilon\,(0 < \varepsilon \ll 1)$，定义正定矩阵

$$P_\varepsilon = \begin{bmatrix} X_\varepsilon & P_{12} \\ P_{12} & L_\varepsilon \end{bmatrix},$$

其中

$$X_\varepsilon = \begin{bmatrix} X_{11} & \varepsilon X_{21}^T \\ X_{21} & X_{22} \end{bmatrix}, \quad P_{12} = \begin{bmatrix} P_{121} & 0 \\ 0 & P_{122} \end{bmatrix}, \quad L_\varepsilon = \begin{bmatrix} L_{11} & \varepsilon L_{21}^T \\ L_{21} & L_{22} \end{bmatrix},$$

且$X_{11} \in \mathbf{R}^{n\times n}$，$X_{22} \in \mathbf{R}^{m\times m}$，$L_{11} \in \mathbf{R}^{n\times n}$，$L_{22} \in \mathbf{R}^{m\times m}$分别为对称正定矩阵，$X_{21} \in \mathbf{R}^{m\times n}$，$L_{21} \in \mathbf{R}^{m\times n}$为任意矩阵，$P_{121} \in \mathbf{R}^{n\times n}$，$P_{122} \in \mathbf{R}^{m\times m}$为对称矩阵，则存在$\varepsilon_0^* > 0$，对于任意的$\varepsilon \in (0, \varepsilon_0^*]$，矩阵$P_\varepsilon$满足$E_{cl}(\varepsilon)P_\varepsilon = P_\varepsilon^T E_{cl}(\varepsilon) > 0$。

可构造Lyapunov泛函

$$V(\tilde{x}(t)) = \tilde{x}^T(t) E_{cl}(\varepsilon) P_\varepsilon \tilde{x}(t) + \int_{t-\tau(t)}^{t} \tilde{x}^T(\sigma) Q \tilde{x}(\sigma) \mathrm{d}\sigma > 0,$$

其中

$$Q = \begin{bmatrix} Q_{11} & 0 \\ 0 & Q_{22} \end{bmatrix},$$

$Q_{11} \in \mathbf{R}^{(n+m)\times(n+m)}$，$Q_{22} \in \mathbf{R}^{(n+m)\times(n+m)}$ 分别为对称正定矩阵。

$$\begin{aligned}\dot{V}(\tilde{x}(t)) &= \dot{\tilde{x}}^T(t)E_{cl}(\varepsilon)P_\varepsilon\tilde{x}(t) + \tilde{x}^T(t)E_{cl}(\varepsilon)P_\varepsilon\dot{\tilde{x}}(t) \\ &\quad + \tilde{x}^T(t)Q\tilde{x}(t) - (1-\dot{\tau}(t))\tilde{x}^T(t-\tau(t))Q\tilde{x}(t-\tau(t)) \\ &\le \tilde{x}^T(t)\left[A_{cl}^T(\mu)P_\varepsilon + P_\varepsilon^T A_{cl}(\mu) + Q\right]\tilde{x}(t) \\ &\quad + \tilde{x}^T(t-\tau(t))A_{dcl}^T(\mu)P_\varepsilon\tilde{x}(t) \quad + w^T(t)B_{cl}^T(\mu)P_\varepsilon\tilde{x}(t) \\ &\quad + \tilde{x}^T(t)P_\varepsilon^T A_{dcl}(\mu)\tilde{x}(t-\tau(t)) + \tilde{x}^T(t)P_\varepsilon^T B_{cl}(\mu)w(t) \\ &\quad - (1-\beta)\tilde{x}^T(t-\tau(t))Q\tilde{x}(t-\tau(t)) - w^T(t)w(t) + w^T(t)w(t).\end{aligned}$$

在不等式两边分别加 $\gamma^{-2}\tilde{z}^T(t)\tilde{z}(t)$ ($\gamma > 0$ 为任意给定常数)，然后对于任意 $T > 0$，不等式两边积分得

$$\begin{aligned}&V(\tilde{x}(T)) + \gamma^{-2}\int_0^T \tilde{z}^T(t)\tilde{z}(t)\mathrm{d}t \\ &\le \int_0^T \begin{bmatrix}\tilde{x}(t)\\ w(t)\\ \tilde{x}(t-\tau(t))\end{bmatrix}^T \begin{bmatrix} A_{cl}^T(\mu)P_\varepsilon + P_\varepsilon^T A_{cl}(\mu) + Q + \gamma^{-2}C_{cl}^T(\mu)C_{cl}(\mu) & * & * \\ B_{cl}^T(\mu)P_\varepsilon & -I & * \\ A_{dcl}^T(\mu)P_\varepsilon & 0 & -(1-\beta)Q\end{bmatrix} \\ &\quad \begin{bmatrix}\tilde{x}(t)\\ w(t)\\ \tilde{x}(t-\tau(t))\end{bmatrix}\mathrm{d}t + \int_0^T w^T(t)w(t)\mathrm{d}t.\end{aligned}$$

$V(\tilde{x}(T)) > 0$。因此，当且仅当 P_ε 满足不等式 (11) 时

$$\int_0^T \tilde{z}^T(t)\tilde{z}(t)\mathrm{d}t \le \gamma^2\int_0^T w^T(t)w(t)\mathrm{d}t \le \gamma^2\int_0^\infty w^T(t)w(t)\mathrm{d}t$$

成立。我们有

$$\begin{bmatrix} A_{cl}^T(\mu)P_\varepsilon + P_\varepsilon^T A_{cl}(\mu) + Q + \gamma^{-2}C_{cl}^T(\mu)C_{cl}(\mu) & * & * \\ B_{cl}^T(\mu)P_\varepsilon & -I & * \\ A_{dcl}^T(\mu)P_\varepsilon & 0 & -(1-\beta)Q\end{bmatrix} < 0. \tag{11}$$

显然，不等式 (11) 可改写为

$$\begin{bmatrix} A_{cl}^T(\mu)P + P^T A_{cl}(\mu) + Q + \gamma^{-2}C_{cl}^T(\mu)C_{cl}(\mu) & * & * \\ B_{cl}^T(\mu)P & -I & * \\ A_{dcl}^T(\mu)P & 0 & -(1-\beta)Q\end{bmatrix} + O(\varepsilon) < 0, \tag{12}$$

其中

$$P=\begin{bmatrix} X & P_{12} \\ P_{12} & L \end{bmatrix}, \quad X=\begin{bmatrix} X_{11} & 0 \\ X_{21} & X_{22} \end{bmatrix},$$

$$L=\begin{bmatrix} L_{11} & 0 \\ L_{21} & L_{22} \end{bmatrix}, \quad P_{12}=\begin{bmatrix} P_{121} & 0 \\ 0 & P_{122} \end{bmatrix},$$

$X_{11}\in\mathbf{R}^{n\times n}$, $X_{22}\in\mathbf{R}^{m\times m}$, $L_{11}\in\mathbf{R}^{n\times n}$, $L_{22}\in\mathbf{R}^{m\times m}$ 为对称正定矩阵，$P_{121}\in\mathbf{R}^{n\times n}$, $P_{122}\in\mathbf{R}^{m\times m}$ 为对称矩阵。

根据文献 [18,19] 中的结论，可得存在于 $\varepsilon_1^*>0$，对于任意的 $\varepsilon\in(0,\varepsilon_1^*]$，当不等式 (13) 成立时，不等式 (12) 成立。下面利用

$$\begin{bmatrix} A_{cl}^T(\mu)P+P^TA_{cl}(\mu)+Q+\gamma^{-2}C_{cl}^T(\mu)C_{cl}(\mu) & * & * \\ B_{cl}^T(\mu)P & -I & * \\ A_{dcl}^T(\mu)P & 0 & -(1-\beta)Q \end{bmatrix}<0, \tag{13}$$

对不等式 (13) 应用 Schur 补定理得

$$\begin{bmatrix} A_{cl}^T(\mu)P+P^TA_{cl}(\mu)+Q & * & * & * \\ B_{cl}^T(\mu)P & -I & * & * \\ C_{cl}(\mu) & 0 & -\gamma^2 I & * \\ A_{dcl}^T(\mu)P & 0 & 0 & -(1-\beta)Q \end{bmatrix}<0, \tag{14}$$

其中

$$P=\begin{bmatrix} X & P_{12} \\ P_{12} & L \end{bmatrix}.$$

令

$$P^{-1}=\begin{bmatrix} Z & S_{21} \\ S_{12} & S_{22} \end{bmatrix},$$

则

$$P\begin{bmatrix} Z & I \\ S_{12} & 0 \end{bmatrix}=\begin{bmatrix} I & X \\ 0 & P_{12} \end{bmatrix},$$

其中

$$Z=\begin{bmatrix} Z_{11} & 0 \\ Z_{21} & Z_{22} \end{bmatrix},$$

$Z_{11}\in\mathbf{R}^{n\times n}$, $Z_{22}\in\mathbf{R}^{m\times m}$ 为对称正定矩阵，

$$S_{12}=\begin{bmatrix} S_{121} & 0 \\ S_{122} & S_{123} \end{bmatrix},$$

$S_{121} \in \mathbf{R}^{n\times n}$，$S_{123} \in \mathbf{R}^{m\times m}$ 为任意阵。

令

$$E_1 = \begin{bmatrix} Z & I \\ S_{12} & 0 \end{bmatrix}, \quad E_2 = \begin{bmatrix} I & X \\ 0 & P_{12} \end{bmatrix}.$$

对不等式 (14) 左边的矩阵分别左乘矩阵 $\mathrm{diag}\{E_1^T, I, I, I\}$ 和右乘矩阵 $\mathrm{diag}\{E_1, I, I, I\}$，并结合式 (5)，得

$$\begin{bmatrix} \Psi_{11} & * & * & * & * & * \\ \Psi_{21} & \Psi_{22} & * & * & * & * \\ B_1^T(\mu) & \Psi_{32} & -I & * & * & * \\ C_1(\mu)Z - C_f(\mu)S_{12} & C_1(\mu) & 0 & -\gamma^2 I & * & * \\ A_d^T(\mu) & A_d^T(\mu)X & 0 & 0 & -(1-\beta)Q_{11} & * \\ 0 & 0 & 0 & 0 & 0 & -(1-\beta)Q_{22} \end{bmatrix} < 0, \tag{15}$$

其中

$$\Psi_{11} = Z^T A^T(\mu) + A(\mu)Z + Z^T Q_{11} Z + S_{12}^T Q_{22} S_{12},$$

$$\Psi_{21} = A^T(\mu) + X^T A(\mu)Z + P_{12}^T B_f(\mu)C_2(\mu)Z + P_{12}^T A_f(\mu)S_{12} + Q_{11}Z,$$

$$\Psi_{22} = A^T(\mu)X + X^T A(\mu) + C_2^T(\mu)B_f^T(\mu)P_{12} + P_{12}^T B_f(\mu)C_2(\mu) + Q_{11},$$

$$\Psi_{32} = B_1^T(\mu)X + D_{21}^T(\mu)B_f^T(\mu)P_{12}.$$

令

$$J(\mu) = P_{12}^T B_f(\mu), \quad \tilde{J}(\mu) = C_f(\mu)S_{12}, \quad \hat{J}(\mu) = P_{12}^T A_f(\mu)S_{12}, \tag{16}$$

则矩阵不等式 (15) 可改写为

$$\begin{bmatrix} \Gamma_{11} & * & * & * & * & * \\ \Gamma_{21} & \Gamma_{22} & * & * & * & * \\ B_1^T(\mu) & \Gamma_{32} & -I & * & * & * \\ C_1(\mu)Z - \tilde{J}(\mu) & C_1(\mu) & 0 & -\gamma^2 I & * & * \\ A_d^T(\mu) & A_d^T(\mu)X & 0 & 0 & -(1-\beta)Q_{11} & * \\ 0 & 0 & 0 & 0 & 0 & -(1-\beta)Q_{22} \end{bmatrix} < 0, \tag{17}$$

其中

$$\Gamma_{11} = Z^T A^T(\mu) + A(\mu)Z + Z^T Q_{11} Z + S_{12}^T Q_{22} S_{12},$$

$$\Gamma_{21} = A^T(\mu) + X^T A(\mu)Z + J(\mu)C_2(\mu)Z + \hat{J}(\mu) + Q_{11}Z,$$

$$\Gamma_{22} = A^T(\mu)X + X^T A(\mu) + C_2^T(\mu)J^T(\mu) + J(\mu)C_2(\mu) + Q_{11},$$

$$\Gamma_{32} = B_1^T(\mu)X + D_{21}^T(\mu)J^T(\mu).$$

对不等式(17)左边的矩阵，分别左乘矩阵 $\mathrm{diag}\{Z^{-T}, I, I, I, I, Z^{-T}S_{12}^T\}$ 和右乘矩阵 $\mathrm{diag}\{Z^{-1}, I, I, I, I, S_{12}Z^{-1}\}$，并对所得不等式令

$$Z^{-1}=Y,\quad M(\mu)=\hat{J}(\mu)Y,\quad N(\mu)=\tilde{J}(\mu)Y,\quad \hat{Q}_{22}=Y^TS_{12}^TQ_{22}S_{12}Y,$$

可得下列不等式

$$\begin{bmatrix} \Theta_{11} & * & * & * & * & * \\ \Theta_{21} & \Theta_{22} & * & * & * & * \\ B_1^T(\mu)Y & \Theta_{32} & -I & * & * & * \\ C_1(\mu)-N(\mu) & C_1(\mu) & 0 & -\gamma^2 I & * & * \\ A_d^T(\mu) & A_d^T(\mu)X & 0 & 0 & -(1-\beta)Q_{11} & * \\ 0 & 0 & 0 & 0 & 0 & -(1-\beta)\hat{Q}_{22} \end{bmatrix}<0, \tag{18}$$

其中

$$\Theta_{11}=Y^TA(\mu)+A^T(\mu)Y+Q_{11}+\hat{Q}_{22},$$

$$\Theta_{21}=A^T(\mu)Y+X^TA(\mu)+J(\mu)C_2(\mu)+M(\mu)+Q_{11},$$

$$\Theta_{22}=A^T(\mu)X+X^TA(\mu)+C_2^T(\mu)J^T(\mu)+J(\mu)C_2(\mu)+Q_{11},$$

$$\Theta_{32}=B_1^T(\mu)X+D_{21}^T(\mu)J^T(\mu).$$

由式(16)得

$$A_f(\mu)=P_{12}^{-T}\hat{J}(\mu)S_{12}^{-1},\quad B_f(\mu)=P_{12}^{-T}J(\mu),\quad C_f(\mu)=\tilde{J}(\mu)S_{12}^{-1},$$

因此，滤波器(4)的传函矩阵为

$$\begin{aligned} H_{\hat{z}y}(s) &= \tilde{J}(\mu)S_{12}^{-1}\left[sI-P_{12}^{-T}\hat{J}(\mu)S_{12}^{-1}\right]^{-1}P_{12}^{-T}J(\mu) \\ &= \tilde{J}(\mu)\left[s(I-XZ)-\hat{J}(\mu)\right]^{-1}J(\mu) \\ &= N(\mu)\left[sI-(Y-X)^{-1}M(\mu)\right]^{-1}(Y-X)^{-1}J(\mu), \end{aligned}$$

而且

$$A_f(\mu)=(Y-X)^{-1}M(\mu),\quad B_f(\mu)=(Y-X)^{-1}J(\mu),\quad C_f(\mu)=N(\mu).$$

结合式(2)、(4)，将不等式(18)改写为

$$\sum_{i=1}^{r}\mu_i^2\Pi_{ii}+\sum_{\substack{i,j=1\\ i<j}}^{r}\mu_i\mu_j\left(\Pi_{ij}+\Pi_{ji}\right)<0, \tag{19}$$

因此，存在 $\varepsilon^*>0$，$\varepsilon^*=\min\{\varepsilon_0^*,\varepsilon_1^*\}$，对于任意的 $\varepsilon\in(0,\varepsilon^*]$，线性矩阵不等式(6),(7)成立。

另外，由 $E_{cl}(\varepsilon)P_\varepsilon > 0$，$E(\varepsilon)X_\varepsilon = X_\varepsilon^T E(\varepsilon)$ 可得

$$\begin{aligned} & E_1^T E_{cl}(\varepsilon)P_\varepsilon E_1 \\ = & \begin{bmatrix} Z^T E(\varepsilon)(XZ + P_{12}S_{12}) + S_{12}^T E(\varepsilon)(P_{12}Z + LS_{12}) & (Z^T X^T + S_{12}^T P_{12})E(\varepsilon) \\ E(\varepsilon)(XZ + P_{12}S_{12}) & X^T E(\varepsilon) \end{bmatrix} \\ & + O(\varepsilon) > 0. \end{aligned} \tag{20}$$

根据 $PP^{-1} = I$ 和文献 [18,19] 的结论，可知存在 $\varepsilon_2^* > 0$，对于任意的 $\varepsilon \in (0, \varepsilon_2^*]$，当不等式 (21) 成立时，不等式 (20) 成立。注意到

$$\begin{bmatrix} Z^T E(\varepsilon) & E(\varepsilon) \\ E(\varepsilon) & X^T E(\varepsilon) \end{bmatrix} > 0, \tag{21}$$

应用 Schur 补定理，并将 $Y = Z^{-1}$ 带入得：Y_{11} 和 Y_{22} 为对称正定矩阵，且不等式 (8)、(9) 成立。

因此，存在 $\varepsilon^* > 0$，$\varepsilon^* = \min\{\varepsilon_0^*, \varepsilon_1^*, \varepsilon_2^*\}$，对于任意的 $\varepsilon \in (0, \varepsilon^*]$，闭环系统 (5) 满足给定 H_∞ 性能指标 γ。

当 $w(t) \equiv 0$ 时，有

$$\dot V(\tilde x(t)) \le \begin{bmatrix} \tilde x(t) \\ \tilde x(t - \tau(t)) \end{bmatrix}^T \begin{bmatrix} A_{cl}^T(\mu)P_\varepsilon + P_\varepsilon^T A_{cl}(\mu) + Q & * \\ A_{dcl}^T(\mu)P_\varepsilon & -(1-\beta)Q \end{bmatrix} \begin{bmatrix} \tilde x(t) \\ \tilde x(t - \tau(t)) \end{bmatrix}.$$

根据文献 [18,19] 的结论和不等式 (14)，得 $\dot V(\tilde x(t)) < 0$，闭环系统 (5) 渐进稳定。

矩阵不等式 (6)-(9)是与 ε 无关的线性矩阵不等式组，因此，可以应用Matlab 的 LMI 工具箱中的求解器 feasp 求解。若有可行解，根据定理1 可构造系统 (2) 的一个 γ-次优 H_∞ 滤波器。考虑求解如下优化问题，如果该问题有解，则结合定理1，利用其最优解可以得到系统(2)的最优 H_∞ 滤波器。本文应用 LMI 中的求解器 mincx 求解。

$$\min\ \gamma \quad \text{s.t.} \quad (6)\text{-}(9). \tag{22}$$

4 仿真例子

考虑由如下两条规则描述的模糊奇异摄动系统

R1：If $x_1(t)$ is F_1, then

$$\begin{aligned} E(\varepsilon)\dot x(t) &= A_1 x(t) + A_{d1} x(t - \tau(t)) + B_{11} w(t), \\ z(t) &= C_{11} x(t), \\ y(t) &= C_{21} x(t) + D_{211} w(t). \end{aligned} \tag{23}$$

R1：If $x_1(t)$ is F_2, then

$$\begin{aligned} E(\varepsilon)\dot x(t) &= A_2 x(t) + A_{d2} x(t - \tau(t)) + B_{12} w(t), \\ z(t) &= C_{12} x(t), \\ y(t) &= C_{22} x(t) + D_{212} w(t), \end{aligned} \tag{24}$$

其中

$$A_1=\begin{bmatrix}-0.1 & 10\\ -1 & -1\end{bmatrix},\quad A_2=\begin{bmatrix}-4.6 & 10\\ -1 & -1\end{bmatrix},\quad A_{d1}=\begin{bmatrix}-0.1 & 0.2\\ -0.1 & -0.1\end{bmatrix},$$

$$E(\varepsilon)=\begin{bmatrix}1 & 0\\ 0 & \varepsilon\end{bmatrix},\quad A_{d2}=\begin{bmatrix}-0.4 & 1.2\\ -0.1 & -0.1\end{bmatrix},\quad B_{11}=B_{12}=\begin{bmatrix}0\\ 0.1\end{bmatrix},$$

$$C_{11}=C_{12}=C_{21}=C_{22}=\begin{bmatrix}1 & 0\end{bmatrix},\quad D_{211}=D_{212}=0.1,\quad \tau(t)=0.5t.$$

对应模糊集合的隶属度函数为

$$\mu_1(x_1(t))=0.5x_1^2(t),\quad \mu_2(x_1(t))=1-\mu_1(x_1(t)). \tag{25}$$

应用LMI工具箱及其求解器mincx，求解式(22)，得到最小扰动参数$\gamma=4$时，实例系统最优H_∞滤波器参数为

$$A_{f1}=\begin{bmatrix}1.0036 & 3.3433\\ 2.2423 & -25.6991\end{bmatrix},\quad A_{f2}=\begin{bmatrix}-58.1831 & -6.8660\\ -104.7622 & -48.7340\end{bmatrix},\quad B_{f1}=\begin{bmatrix}0.5463\\ 1.1486\end{bmatrix},$$

$$B_{f2}=\begin{bmatrix}9.0520\\ 20.2301\end{bmatrix},\quad C_{f1}=\begin{bmatrix}-1.3364 & -0.3322\end{bmatrix},\quad C_{f2}=\begin{bmatrix}-22.1426 & -12.4140\end{bmatrix}.$$

以初始条件$x(0)=[1,0,0,0]^T$, $w(t)=\sin(\pi t)e^{-0.1t}$，进行仿真，图1至图4为对应$\varepsilon=0.001$, $\varepsilon=0.1$时的闭环系统状态响应。可以发现对充分小的ε，系统能够获得满意的性能。

注：图1至图4中的x1, x2, xhat1和xhat2分别表示闭环系统状态变量$x_1(t)$, $x_2(t)$, $\hat{x}_1(t)$, $\hat{x}_2(t)$。

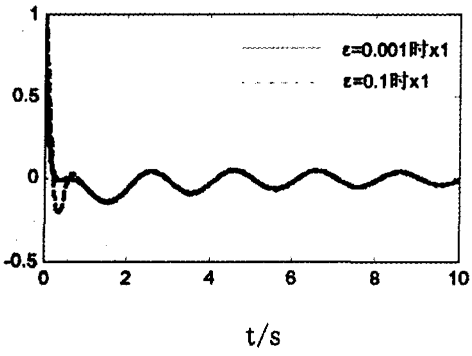

图1: 闭环系统状态变量$x_1(t)$

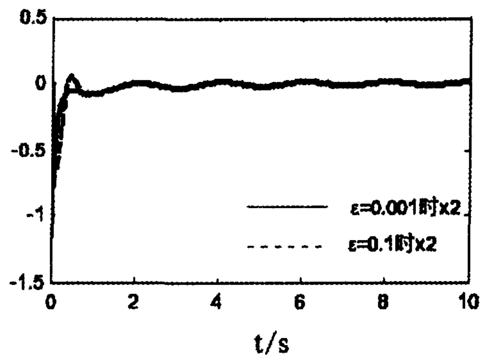

图2: 闭环系统状态变量$x_2(t)$

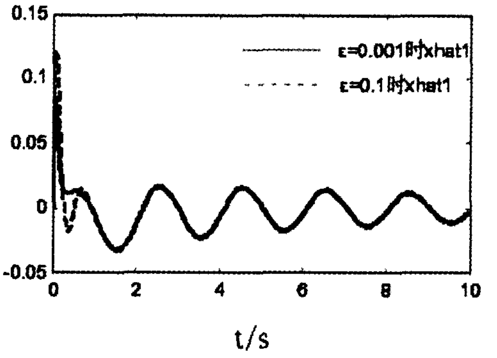

图 3: 闭环系统状态变量$\hat{x}_1(t)$

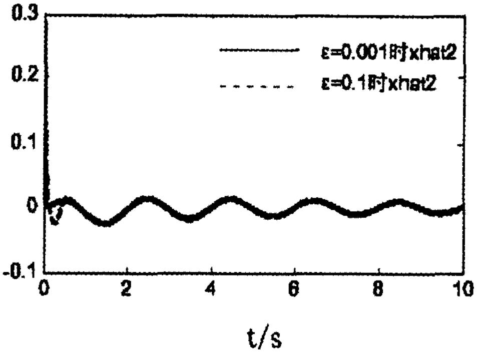

图 4: 闭环系统状态变量$\hat{x}_2(t)$

5 结论

本文建立了时滞模糊奇异摄动系统模型，通过Lyapunov方法和Schur补定理，得到了一类时滞模糊奇异摄动系统存在模糊 H_∞ 滤波器的充分条件。将控制器设计归结为与摄动参数无关的LMIs求解，避免了数值求解过程中的病态问题，并能够获得稳定的数值解。对充分小的ε，所得滤波器使闭环时滞模糊奇异摄动系统渐进稳定，且具有给定的 H_∞ 性能指标。设计方法无需实现快慢分解且适用于标准和非标准非线性时滞奇异摄动系统。

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H_∞ Filtering for Fuzzy Singularly Perturbed Systems with Time-delay

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Abstract: A modeling approach based on Takagi-Sugeno (T-S) is proposed and a fuzzy H_∞ filter is developed for a class of nonlinear singularly perturbed systems with time-varying delay in states. The gains of the H_∞ filter are obtained by solving a set of the ε-independent linear matrix inequalities such that the ill-conditioned problem caused by the interaction of slow and fast dynamic modes is avoided effectively. The designed filter guarantees that the closed-loop fuzzy SPSs system is asymptotically stable and the prescribed level of disturbance attenuation is satisfied for sufficiently small ε. The proposed approach can be applied to both standard and nonstandard nonlinear singularly perturbed systems. A simulation example is provided to illustrate the designed procedures.

Keywords: singular perturbation; time delay; H_∞ filtering; linear matrix inequalities (LMIs); T-S model

Received: 23 Apr 2008. **Accepted:** 05 Dec 2008.

Foundation item: The National High-tech Industrialization Project of China (2005-1).